Trust in the Firm

Some Remarks on the Mathematical Economics of Trust¹

by

Horst Albach²

Abstract:

The paper reports on various mathematical methods which have been used to demonstrate that trust and loyalty in the firm are important economically. Special emphasis is laid on one-stage and two-stage principal-agent-models. They show that trust and loyalty in the firm are strengthened when wages are contracted under relational law (long-term contracts). Another approach to trust in the firm is the prisoner’s dilemma game with an augmented utility function based on the writings by Aristotle and Adam Smith. With Smith’s “fellow-feeling” in the objective function the players reach first-best solutions.

A. Introduction

This paper was written for the CASiM Conference 2012 “The Role of Trust in Business Economics” on June 28, 2012. The conference wants to clarify an important issue for theorists and for practitioners in management: What is the type of theory of the firm for the 21st century? What are the methods that will empower managers to master the problems of firms in the 21st century? The answers to these questions will be manifold. Some may even think that there are other questions more relevant for business in the 21st century than trust.

It may be useful to first find out what others think of the future of management. Therefore let us first look at the articles in “Zeitschrift für Betriebswirtschaft, volume 81 (2011)” and at the McKinsey Quarterly.

The Zeitschrift für Betriebswirtschaft gives a good overview of the methodology that may be helpful for managers to solve the problems of the 21st century. ZfB published 47 articles in the monthly editions and 33 articles in the special editions. Ten of them are empirical studies, two

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accounting studies, one deals with real options, and one each are simulation, game theory and operations research models. 64 of the 80 articles are special studies without making use of a more general methodology. I conclude: “empirical studies are in”. None of them discusses ways of reproducing the data set. Yet, managers may question the results of the empirical studies anyway. They may not hold in their own firms, and they may not hold in the future.

The McKinseyQuarterly reports on a wide range of topics important for managers. 31 issues from 2004 to 2012 form the basis for this overview. Six are devoted to different countries (China 3, India 1, Latin America 1, and Asia 1); five deal with problems in the global world, five discuss strategy, and three deal with government relationships. There are two issues on innovation, two on marketing. McKinsey has published one issue each on growth, on performance, on women and leadership, on climate change, and technology. The issues on health care and on “The 21st-century organization” are particularly interesting.

The conclusions drawn on this basis is:

1. There is a wide gap in the field of dynamic models of the firm. The Center for Advanced Studies in Management (CASiM) at HHL Leipzig Graduate School of Managementshould fill it.
2. There is a lot of interest in empirical research. CASiM should develop its own data base for long-run empirical tests of models of the firm. The institute has to make sure that the data can be checked, repeated, and are time-consistent.
3. Game Theory has become a most important research tool for studying strategic behavior of firms on narrow oligopolistic markets. CASiM should make significant contributions to advancing this method.
4. The subject “Management of Innovation” is still very relevant.
5. “Strategy and Organization” has become very important for multinational companies.
6. CASiM will have to develop dynamic models of the firm and for developing models of the firm in chaotic and turbulent environments.
7. Comparative studies of behavior of firms from different cultures will help managers to remain successful in the 21st century. This includes studies of corporate ethics.

In what follows I will present my own vision of a “Theory of Management in the 21st Century”.
B. Management Theory in the 21st Century

What is management in the 21st century? This was the topic of the first session of the CASiM Conference 2012 “The Role of Trust in Business Economics” on June 28, 2012. I will first mention three areas that management will have to cope with:

- Structural change in the global economy
- Uncertainty in the form of turbulence
- Complexity

I. Structural Change

Structural change is defined by great changes in the flow of goods, services, and money between countries: China, India, Brazil, Europe, the United States, and the struggling countries particularly in Africa.

II. Turbulence

Turbulence is characterized by hardly predictable changes in the environments of firms including conflicts between ancient cultures, between ethnical groups and between radical groups with a religious background, and probably increasing conflicts between the “haves” and the “have-nots” in the world. Water may become a source of serious conflict.\(^3\)

III. Complexity

Complexity is a catchword with many meanings. I define complexity by interdependence of the functional units of the firm and of the firm and its stakeholders.

\(^3\) A special issue of the National Geographic Magazine has dealt with „Water“. See the April 2010 issue on “Water: Our Thirsty World”. 
C. Management Methodology in the 21st Century

What are the tools that managers need to cope with the problems of the 21st century? Most of the managers and consultants today have two answers to this question: resilience and sustainability management. The theorist knows that these words are not new, neither in theory nor in practice. Resilience is innovation management and conservative finance management, and sustainability is going-concern management.

I am convinced that we need hard science and good knowledge in the history of economic thought in order to cope with the problems of the future.

I. Hard Science: Mathematics

By hard science I mean mathematics. Mathematics has been applied in the past for complexity management and will be used for the same purposes in the future. Mathematics specifies clearly the assumptions made, the axioms used, the interdependencies between the variables clearly formulated in equations, and the solutions based on mathematical logic. Mathematics does, however, not produce applicable results. It leads us to the question: Why should the mathematical solution be applicable in managing complexity? In order to answer this question, we need additional tool kits.

II. Empirical tests: Econometrics

One of them is empirical testing of the models against “reality”. Reality, however, is a data set put together from past practical experience. It is a sample of data from the past and sometimes collected in an attempt to prove a hypothesis and most of the time not repeatable. Experimental testing is therefore not without doubt. And very important: Experiments that test hypotheses we already know to be true are as waste of time. Therefore, we need a third tool kit.
III. Plurality of Methods

In this third tool kit we find different mathematical approaches to the problems of management in the future. If they all come to the more or less same conclusion, one may put more trust to their results.

IV. The History of Thought: the Theory of Investment of the Firm

Roughly fifty years ago a fundamental change took place in the theory of the firm. Until then, the static analysis of the inputs of the different production factors into the production function and its single product output had dominated\textsuperscript{4}. Capital was one of the production factors. Why and how the capital stock developed was not analyzed. It was a given. Then, in a comparative production function theory\textsuperscript{5}, changes in the factor inputs were studied, but still in the framework of a static analysis. It became obvious that a truly dynamic analysis of the factor “capital” was needed.

The first steps in the dynamic analysis were devoted to the lifetime of a piece of real investment\textsuperscript{6}. What was the optimal lifetime of a machine? The answer to this question was the basis for determining “the economic rate of depreciation”. With this depreciation rate known one could continue using the old static “cost-benefit-analysis of an investment project” and determining the profitability of the project. However, it was soon recognized that this approach to determining the profitability of pieces of equipment was inadequate. For a short period of time the “internal rate of return” was believed to be the appropriate measure of the profitability of investment. However, this method was seriously flawed also. Finally, the “discounted cash flow method” found general acceptance as a measure of the profitability of an investment project. This method, however, was not without methodological problems. The cash flows from a project could not easily be separated from the total cash flow the firm generated each year. And, secondly, the cash flows from the investment project were uncertain. That caused serious problems: Since the future is uncertain, assumptions had to be

\textsuperscript{4} See, e.g. Schneider, Erich: Das Zeitmoment in der Theorie der Produktion, in: Jahrbücher für Nationalökonomie und Statistik, vol. 143, 1936, pp. 52 and seq.


made about the future\textsuperscript{7}. The future depends on the behavior of the customers and of the behavior of the competitors which were not known. Could management trust the forecasts made by the specialists in the firm? Could management trust the tax policy promises of the government? Hindsight has told managers that forecasts made ten years ago were utterly false. In particular, in an imperfect capital market could management assess the “discount rate” properly? And what about the assumptions about the competitive policy measures of the competitors? We needed a truly dynamic theory of the firm. The first step in this direction was done by Joel Dean. His book “Capital Budgeting”\textsuperscript{8} showed the way, although it was still rooted methodologically in the static price theory of the thirties. It led to a “Dynamic Capital Budget”.\textsuperscript{9} From a large number of projects which were independent of each other, the most profitable ones were selected under the condition that they could be financed by a mixture of short-term and long-term financing with different covenants. Each year management decides on a capital budget which assumes trust between top managers and the investment managers who signal true information. The dynamic capital budget also expresses trust between the firm and its credit institutes who provide the financing means for the capital assets in the annual investment budget. Also, the banks trust the firm that they realize the investment program that forms the basis of their financing contract. The uncertainty of the future was not only inherent in “nature” but also by the competitive acts of competitors in the market and by internal conflicts between management and employees. The theory of investment of the firm had to be extended to a dynamic theory of uncertainty, information, and trust. We will deal with these new developments in the next chapter.

**D. Examples of Management Methodology in the 21\textsuperscript{st} Century**

**I. The Production Function**

The classical theory of the firm is based on what is called a production function. It used to be static. The neo-classical approach is the dynamic production function. It models not only the optimal input of resources for a given output in one period, but it models also the adaptation

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\textsuperscript{8} Dean, Joel: Capital Budgeting, 3rd edition, New York 1951

process that takes place after a change in the environment or within the firm. The dynamic production function then tells management how to optimize the time path to the next equilibrium (which may never be reached when the changes in the environment take place faster than management had planned to be the optimal path). Stefan Kayser\textsuperscript{10} will present a paper on a dynamic production function empirically tested against the transformation processes of Hungary and Poland.

\textbf{II. The Neo-Classical Model of the Firm}

The neo-classical model of the firm may also be used to describe growth and decline processes of the firm in a multi-equation-model with an equation each for the different functions in the firm: production, investment, marketing, procurement, inventory management, and research and development\textsuperscript{11}. It was shown on the basis of the annual financial reports of 295 German and 104 Japanese firms that each manager of a special function in the firm tries to maximize its individual objective function using OR-optimization methods. The result is then communicated to head office which coordinates the partial plans in a step-by-step process until the global optimum is reached. Management trusts the division heads. They report true information. At AEG Corporation it did not hold true. AEG is a good example for the absence of trust and the presence of misinformation. It finally resulted in the insolvency of AEG.

\textbf{III. Control Theory}

Control Theory is a step forward from the theory of variations. It helps to optimize dynamic processes with a fixed starting point and a variable end point. Bertil Näslund\textsuperscript{12} was the first to apply control theory to growth processes of the firm. Thomas Ludwig improved the Näslund model\textsuperscript{13}. He showed that the optimal path into the future of the firm might require a “growth intermission” in order to adjust the real growth path to the optimal path of development in the finance and the capital structure. Such a growth process dominates a path of steady growth. It

\textsuperscript{10} See also: Kayser, Stefan: Die Dynamik der Unternehmenstransformation in Osteuropa, Wiesbaden 1999
\textsuperscript{13} See Ludwig, Thomas: Optimale Expansionspfade der Unternehmung, in: Beiträge zur betriebswirtschaftlichen Forschung Band 49, Wiesbaden 1978
may be better, however, to follow a “steady growth strategy” if stakeholders would lose trust in the firm when it stops growing for a while, or: put otherwise: The manager have to manage trust with their stakeholders so that they do not lose trust in the future of the firm

**IV. Chaos Theory**

Another mathematical description of dynamic processes in the firm is chaos theory. Andreas Pinkwart has analyzed the dynamics of firm growth by a single quadratic equation with lagged variables\(^1\)\(^\text{4}\). Given a certain set of starting data which describe the present situation of the firm, growing may take a chaotic path into the future. The firm may then get into a situation where any decision management takes will be wrong. Management for sustainability means then managing a going concern by careful controlling the data of the present situation and keeping the development of the firm far enough away from the data that might start a chaotic development. This is a new form of controlling the financial statements of the firm. Otherwise, everybody, employees of the firm as well as stakeholders, lose their trust in the top managers.

**V. Game Theory**

Game theory has become an important tool in the analysis of the behavior of small groups of economic agents. Behavior may be cooperative and non-cooperative, short-term or long-term. Game theory helps us to understand more deeply the importance of trust in the firm. The papers by Silke Bustamante and by Michael Tröge analyze trust and aggressiveness in the firm and trust of customers and their credit institutes with game-theoretical methods.

Since in the long-run continuous games with infinite horizons may have many equilibria, evolutionary game theory with experimental setups has become important for the selection of approximately optimal strategies in practice. Fudenberg (1990) analyzes how “cooperation might arise in a population of self-interested agents. If the game is played only once, the unique equilibrium results in inefficient pairs. If, instead, the game is repeated infinitely often and the players are not too inefficient, there are cooperative equilibria in which both players are always cooperative (or put in the words of the author: always trust each other)”. Dal Bó (2005) in his study of infinitely repeated games comes to the conclusion that “repeated

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\(^{1}\) Pinkwart, Andreas: Chaos und Unternehmenskrise, in: Beiträge zur betriebswirtschaftlichen Forschung, Band 69, Wiesbaden 1992
interaction may enable punishment and reward schemes that prevent or limit opportunistic behavior and support cooperation. The danger of retaliation for defection of the other player (“the Shadow of the Future”) significantly reduces opportunistic behavior” (p. 1602). This means in practice that the longer the time of office of the top manager, the greater the trust between top executives. In 2011 Dal Bó and Fréchette presented new experimental evidence on the evolution of cooperation and trust in infinitely repeated games, and in April 2012 Fudenberg and colleagues studied continuous games with learning from experience and continuation probabilities derived from such learning. The players may be “lenient” or in other words ready to consider defection of the partner as an “error” or they may be willing to forgive purposeful defection. The experimental results show that “in an uncertain world, it can be payoff-maximizing to be slow to anger and fast to forgive” (p. 742). Trust is, therefore, a very important type of behavior in a turbulent world.

The really interesting result of this school of thought is that only in continuous games cooperation (trust) will arise, but might be broken any time when too many players are involved. In the one-shot games behavior will always be non-cooperative. The players behave opportunistically, and trust cannot be expected. In the one-shot prisoner’s dilemma discussed below I will show that trust is nevertheless an efficient strategy for the players.

Wilhelm Krelle in his paper “Ethik lohnt sich auch ökonomisch“ developed a non-cooperative two-person-game with perfect information. He studies a “personality-based game”. The results of this game are: “Decent behavior and punishment for deceit are types of behavior that are profitable not only in the long-run, but also in the short-run, if the players can trust each other. In the realm of ethics, there is a sort of “invisible hand” which keeps mankind from degenerating into a gang of cheaters and bandits“.

**VI. Principal-Agent-Theory**

Principal-Agent-Theory is a special form of game theory. In its original form the model assumed a one-period relationship between a principal (P) and an agent (A) who each maximize their profit or utility respectively. They have asymmetric information about their character, action, and information. This gives rise to the “moral hazard”-problem. The two try

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15 Slow to react with punishment in anger
17Krelle, Wilhelm: cf. page 14
to exploit each other and try to avoid this by contracts that reduce the probability of being exploited. This is expensive. By writing “optimal contracts”, people try to overcome their basic fear and distrust. Homo homini lupus! But there are no “optimal contracts” in a world of uncertainty, turbulence, and chaos.

If the contract is what Williamson has called “relational law”, then the optimal contract may avoid moral hazard. If the principal, who offers the contract, trusts his agent, then it is optimal for the agent to honor this evidence of trust and to behave as a loyal agent. This is the dynamic principal-agent-model of Thomas Petersen. I will show the Petersen model in more detail below\textsuperscript{18}.

Petersen has also developed a three-stage-PA-model with a principal, a manager, and a worker. The manager can observe the worker and reports his information to the principal. Petersen shows that such a hierarchy improves trust of the principal in his subordinates on the one side, and loyalty of the worker to the manager on the other. This model was applied to the three-step-situation of shareholders, boards, and executives with striking results. I will give a short description of that model later on.

\textit{VII. The Prisoner’s Dilemma}

The prisoner’s dilemma has become a standard model for a non-cooperative two-person game. Each player maximizes his own utility. The first-best solution of this game cannot be reached because the two players distrust each other. It has always amazed me that theorists should define utility such that the players maximize their utility although they know that they have to go on living with the memory that they have sent their “partner” to the electric chair.

Adam Smith assumed that firms maximize their profits and thereby improve the wealth of nations on the condition that markets are perfect: many suppliers and many customers are coordinated by the “invisible hand” of the market. If, however, the market is imperfect, utility maximization is different from profit maximization. In this case, Adam Smith included “fellow-feeling” in the utility function of each player, and the first-best solution is achieved

because each player trusts his partner. I will present a model of the prisoner’s dilemma with trust later on. It emphasizes the importance of trust in the relationships of members in society.

E. The Petersen-Principal-Agent-Models

1. A one-period-two-persons model

We start with the simplest model of a principal-agent relationship. A principal P has hired an agent A. P offers A a one-period labor contract. The contract maximizes the profits of P. It takes into consideration the objective of the agent to maximize his utility (incentive-compatibility of the contract), and that the wage is higher than any other offer the agent might get from another principal (participation constraint). Thomas Petersen stresses the importance of the incentive compatibility constraint in the labor contract: “the utility function of the agent cannot be influenced by the principal. Or put differently: the preference function of the agent is a tabu for the principal. The personal freedom of the agent has priority and is recognized by the principal as a binding responsibility”\(^{19}\).

The mathematical formulation of this problem is given by equations (1) to (3) below.

P maximizes his profits

\[
\max_{s(x) \in [c,d+x]} \mathbb{E} \left\{ G(x - s(x)) \right\}, \tag{1}
\]

where

\[G = \text{profits of P/utility function of P},\]

\[x = x_{UA, \theta} = \text{total return, where } \theta \text{ models random influences},\]

\[a = \text{performance of A},\]

\[s(x) = \text{part of the total return that is paid to A},\]

\[c = \text{fixed wage},\]

\[E = \text{expected value}.\]

\[
\mathbb{E} \left\{ H \left( s(x), a \right) \right\} \geq \hat{H} \tag{2}
\]

with

\[H = \text{utility function of the agent},\]

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\(^{19}\) Petersen, Thomas, 1989, p. 40
(2) is the participation constraint. The agent is willing to accept the contract only if the utility of subscribing to the contract is higher than the maximum utility he would gain under a contract with another principal.

\[ a \in \arg \max_{a'} \mathbb{E} \{ H(s(x), a') \} . \tag{3} \]

This is the incentive-compatibility constraint. Positive changes in \( a (a') \) lead to higher returns \( x = x(a, \theta) \) and hence to higher \( s(x) \). In general it is assumed that \( P \) is risk-neutral and \( A \) is risk-averse.

The optimal solution of this contract problem is a function \( s(x) \) and not a certain number. \( s(x) \) specifies for all returns what part of the returns the agent will receive. The optimal utility of \( A \) is given by

\[ \hat{s} = \mathbb{E}(\hat{H} + V(a)) . \tag{4} \]

\( V(a) \) is the premium for \( A\)'s performance. It is obvious from (4) that \( A \) receives a wage which is above his reservation wage. \( A \) receives a wage solely based on his results (returns). The principal cannot observe \( A\)'s behavior.

On this basis one can develop models with results control and with behavior control. In the case that \( P \) delegates control to a manager \( M \), we get a Hierarchy-Principal-Agent model. We look at such a model below.

**II. A Two-Stage-Two-Persons-Principal-Agent Model**

We now present the two-stage model. The basic model is simple. Over the years, \( P \) observes the results of the work of \( A \). He infers from these observations information about the behavior of \( A \). The uncertainty of \( A\)'s behavior and of his commitment to \( P \) is continuously reduced, and thus the moral hazard problem becomes less important. Trust in the agent increases. Thomas Petersen has developed such a model.
P has to solve the following problem. He wants to write an optimal wage contract

$$\max_{(s_t, a_t)_{t=1}^2} \mathbb{E} \left\{ \sum_{t=1}^2 G_t(x_t - s_t(x_1, \ell_1, x_{t-1}, \ell_{t-1}) \right\}, \tag{5}$$

where the notation is analogous to the previous section and where $l$ is the signal observed by the two partners. The signal $l$ is an indicator of the loyalty of the agent. The index $t$ ($t=1, 2$) indicates the time period $t$.

The reservation constraint of the agent is

$$\mathbb{E} \left\{ \sum_{t=1}^2 H_t(s_t(x_1, \ell_1, x_t, \ell_t), a_t(x_1, \ell_1, x_{t-1}, \ell_{t-1}) \right\} \geq \tilde{H}. \tag{6}$$

This is the expected utility of the agent which depends on the wage strategy of the principal and the work intensity (commitment) of the agent. One notices that $a_t$ depends on the return $x_t$ in period 1 and on the loyalty information $l_t$ that the principal receives at the end of period 1.

We write the incentive-compatibility constraint

$$\left( a_t \right)_{t=1}^2 \in \arg \max \mathbb{E} \left\{ \sum_{t=1}^2 H_t(s_t(x_1, \ell_1, x_t, \ell_t), a_t(x_1, \ell_1, x_{t-1}, \ell_{t-1}) \right\}. \tag{7}$$

Simplifications lead to the following problem of the principal who wants to offer an optimal labor contract:
\begin{align*}
L(s_1(x_1, \ell_1), s_2(x_1, \ell_1, x_2, \ell_2), a_1, a_2(x_1, \ell_1), \lambda, \mu_1, \mu_2(x_1, \ell_1)) \\
= \int \left[ G_1(x_1 - s_1(x_1, \ell_1)) + \int G_2(x_2 - s_2(x_1, \ell_1, x_2, \ell_2)) f_2(x_2, \ell_2|a_2(x_1, \ell_1)) \right] f_1(x_1, \ell_1|a_1) \\
+ \lambda \left\{ \int \left[ U_1(s_1(x_1, \ell_1)) + \int U_2(s_2(x_1, \ell_1, x_2, \ell_2)) f_2(x_2, \ell_2|a_2(x_1, \ell_1)) - V_2(a_2(x_1, \ell_1)) \right] \\
\cdot f_1(x_1, \ell_1|a_1) - V_1(a_1) - H \right\} \\
+ \mu_1 \left\{ \int \left[ U_1(s_1(x_1, \ell_1)) + \int U_2(s_2(x_1, \ell_1, x_2, \ell_2)) f_2(x_2, \ell_2|a_2(x_1, \ell_1)) - V_2(a_2(x_1, \ell_1)) \right] \\
\cdot f_1''(x_1, \ell_1|a_1) \right\} \\
+ \mu_2(x_1, \ell_1) \left\{ \int U_2(s_2(x_1, \ell_1, x_2, \ell_2)) f_2''(x_2, \ell_2|a_2(x_1, \ell_1)) - V_2''(a_2(x_1, \ell_1)) \right\} f_1(x_1, \ell_1|a_1).
\end{align*}

This is the objective function \( L \) (for Lagrange) of the principal with the Lagrange factor \( \lambda \) for the participation constraint and \( \mu_1 \) and \( \mu_2 \) the Lagrange factors for the incentive compatibility constraint.

Optimizing the Lagrange function gives us the optimal intensity of work for the agent

\begin{align*}
\int \left[ G_1(x_1 - s_1(x_1, \ell_1)) + \int G_2(x_2 - s_2(x_1, \ell_1, x_2, \ell_2)) f_2(x_2, \ell_2|a_2(x_1, \ell_1)) \right] f_1''(x_1, \ell_1|a_1) \\
+ \mu_1 \left\{ \int \left[ U_1(s_1(x_1, \ell_1)) + \int U_2(s_2(x_1, \ell_1, x_2, \ell_2)) f_2(x_2, \ell_2|a_2(x_1, \ell_1)) - V_2(a_2(x_1, \ell_1)) \right] \\
\cdot f_1''(x_1, \ell_1|a_1) - V_1''(a_1) \right\} = 0
\end{align*}

and the optimal wage strategy of the principal

\begin{align*}
\int \int G_2(x_2 - s_2(x_1, \ell_1, x_2, \ell_2)) f_2''(x_2, \ell_2|a_2(x_1, \ell_1)) f_1(x_1, \ell_1|a_1) + \int \mu_2(x_1, \ell_1) \\
\cdot \left\{ \int U_2(s_2(x_1, \ell_1, x_2, \ell_2)) f_2''(x_2, \ell_2|a_2(x_1, \ell_1)) - V_2''(a_2(x_1, \ell_1)) \right\} f_1(x_1, \ell_1|a_1) = 0.
\end{align*}

These equations are used to compute the optimal work intensity of the agent and the optimal wage strategies of the principal. They use the total evidence of previous periods to determine the optimal wage contract. The higher the total return, the higher the premium payment for the agent. With equal returns and an increasing indicator of loyalty the agent is rewarded for his
loyalty. The higher the return of the first period, the higher the expected utility of the agent in period 2.

Petersen concludes: The optimal long-term contract is characterized by the fact that the salary of the agent is smoothed over time. Or, put otherwise: the uncertainty of the first-period return is distributed over the two periods. And, generally speaking, “relational contracts in the sense of Williamson (long-term contracts under relational law) improve the risk distribution between the principal and the agent. They improve the incentive problem for both partners. The longer the agent works for the principal, the better informed is the principal about the loyalty of his agent, and the less an error of the agent weighs in the eyes of the principal. The trust relationship between the principal and the agent is strengthened. Long-term contracts activate the loyalty of the agent to the firm.”

III. Petersen’s Hierarchy Model

Petersen seems to have been one of the first to develop Hierarchy-PA-Models. The model consists of the principal P, the manager M, and the worker A. M observes the work of A and reports his findings of A’s commitment to his work to P. P can neither observe the behavior of M nor that of A.

The model is exemplified for the relationship between shareholder (P), Board of Directors (M), and Executives (A). The result is the following recommendation: The members of the Board receive the smallest compensation possible: it just covers their reservation salary. The executives receive a salary which depends on the profits of the firm and on their behavior as reported in the annual financial reports. The board has no incentive to misinform the shareholders, because the members receive a fixed amount of money. The members of the board always tell the truth.

Thomas Petersen concludes: The government structure of the corporation is an efficient incentive-compatible organization as long as all members of the organization behave in accordance with the law. And generally: a bureaucratic organization is optimal given certain conditions.

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F. A Prisoner’s Dilemma Game with Trust

I now present a one-stage game with trust. The well-known prisoner’s dilemma\textsuperscript{24} is a non-cooperative game in static form. The basic assumption is that the two players each want to maximize their utility, which is defined as their individual “profits”. They behave opportunistically.

Table 1 shows a numerical example of their pay-off matrix.

<table>
<thead>
<tr>
<th>Actions A / B</th>
<th>B1</th>
<th>B2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>-30</td>
<td>-60</td>
</tr>
<tr>
<td></td>
<td>-30</td>
<td>-3</td>
</tr>
<tr>
<td>A2</td>
<td>-50</td>
<td>-4</td>
</tr>
<tr>
<td></td>
<td>-2</td>
<td>-5</td>
</tr>
</tbody>
</table>

The so-called rational solution for both players A and B is to play A1B1. The stable equilibrium of the game is a loss of 30 units of profits for each player. Inspection shows that A2B2 would be a better solution. But it is not stable under the conditions of the non-cooperative game. The temptation to get away with smaller losses (A2: -2; B2: -3) is considered too great.

The reason for considering A1B1 a stable solution is quite obviously the lack of trust between the two partners. But why shouldn’t they trust each other? The lack of trust costs B 26 units of profits, and A 25 units of profits. Obviously, the objective function of the players is wrong. It is without any basis in the history of management thought, and is without any knowledge whatsoever of the history of corporate ethics. Profit maximization is, according to Adam

Smith, a reasonable objective function only if the market is perfect. If it is not, the partner’s well-being has to be added as an element of the objective function. As early as at the times of Aristotle the objective function of an agent in the polis contained the happiness of his fellow-citizens as an element of his own happiness. In the same vein of thinking Adam Smith talked of fellow-feeling.

If player A applies the weight $\alpha_1 = 0.8$ to his own pay-off and the weight $\alpha_2 = 0.2$ to player B’s pay-off, then we get the pay-off for player A shown in table 2, and A2B2 becomes the optimal solution. This means that trust with a weight of only 20% in the utility function of player A is worth 25.2 (= 30 – 4.8) units!

Table 2: Pay-Off Matrix

<table>
<thead>
<tr>
<th>Actions</th>
<th>B1</th>
<th>B2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>-30</td>
<td>-14.4</td>
</tr>
<tr>
<td>A2</td>
<td>-40.4</td>
<td>-4.8</td>
</tr>
</tbody>
</table>

Let us now consider the Aristotle utility function with costs $K(x)$, given by

$$U = \alpha_1 \left( p(x) x - K(x) \right) + \alpha_2 \left( \frac{c x}{P(x,p(x),K(x))} \right) = F(x), \quad \text{(11)}$$

with

- $\alpha_1, \alpha_2 =$ weights of profits $P$ and contribution to fellow-feeling $F$,
- $x =$ production volume,
- $P = P(x,p(x),K(x)) =$ profits,
- $p = p(x) =$ price of the product with production volume $x$,
- $K(x) =$ cost function,
- $F = F(x) = cx =$ fellow feeling, with $c =$ wellness factor of the fellow for each unit of $x$ produced,
\( \alpha_1 + \alpha_2 = 1. \)  \hfill (12)

We assume a symmetric game. Demand is determined by (13)

\[ p(x) = A - B x. \]  \hfill (13)

The cost function is

\[ K(x) = C x + D x^2 + FK, \]  \hfill (14)

with FK = fix costs.

Maximizing (11) with respect to \( x \) we get the optimal output and price

\[ x^* = \frac{A - C}{2 (B + D)} + \frac{\alpha_2}{\alpha_1} \frac{c}{2 (B + D)} \]  \hfill (15)

and

\[ p^* = A - B x^* = A - \frac{B (A - C)}{2 (B + D)} - \frac{\alpha_2}{\alpha_1} \frac{B c}{2 (B + D)}. \]  \hfill (16)

In order to illustrate the solutions let us use a numerical example with \( C = 2, D = 6, c = 4, B = 1, A = 16, \) and FK = 4.

We get for \( \alpha_i = 1 \) the profit maximizing solution \( x^* = 1.00; p^* = 15; U^* = 3.0. \)

With \( \alpha_i = 0.5 \) the Aristotle solution is \( x^* = 1.29; p^* = 14.71; U^* = 3.79. \)

This means: The firm produces more (because it is in the social interest to have more goods on the market, and the firm feels that its social responsibility demands to recognize such interests) at a lower price (which benefits the customers – the firm’s fellow-feeling). Total utility is higher by 0.79 units. The weighted profits of the firm are reduced from 3 to 1.21 units, while the weighted fellow-feeling increases from 0 to 2.57 units. One might argue that this solution takes the interests of the customers (the neighbors in the polis) too seriously. The firm might assign a weight of less than \( \frac{1}{2} \) to the interests of the customers without violating its social responsibility. On the other hand one might argue that under the conditions of markets with perfect competition the firm would make no profits at all.
Table 3 shows the figures for $\alpha_i = 0.3$ to $\alpha_i = 1.0$ increased in steps of 0.1, and figure 1 shows a graphic representation of the total utility $U^*$, the weighted profits $\alpha_i P^*$ and the weighted fellow-feeling $\alpha_i F^*$ based on an interpolation for data for $\alpha_i = 0.3, 0.31, 0.32, \ldots, 0.99, 1.00$.

<table>
<thead>
<tr>
<th>$\alpha_i$ = Weight</th>
<th>$U^*$ = Total Utility</th>
<th>$\alpha_i P^*$ = Weighted Profits</th>
<th>$\alpha_i F^*$ = Weighted Fellow-Feeling</th>
<th>$x^*$ = Production Volume</th>
<th>$p^*$ = Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>3.00</td>
<td>3.00</td>
<td>0.00</td>
<td>1.00</td>
<td>15.00</td>
</tr>
<tr>
<td>0.9</td>
<td>3.10</td>
<td>2.69</td>
<td>0.41</td>
<td>1.03</td>
<td>14.97</td>
</tr>
<tr>
<td>0.8</td>
<td>3.23</td>
<td>2.37</td>
<td>0.86</td>
<td>1.07</td>
<td>14.93</td>
</tr>
<tr>
<td>0.7</td>
<td>3.37</td>
<td>2.03</td>
<td>1.35</td>
<td>1.12</td>
<td>14.88</td>
</tr>
<tr>
<td>0.6</td>
<td>3.55</td>
<td>1.65</td>
<td>1.90</td>
<td>1.19</td>
<td>14.81</td>
</tr>
<tr>
<td>0.5</td>
<td>3.79</td>
<td>1.21</td>
<td>2.57</td>
<td>1.29</td>
<td>14.71</td>
</tr>
<tr>
<td>0.4</td>
<td>4.11</td>
<td>0.69</td>
<td>3.43</td>
<td>1.43</td>
<td>14.57</td>
</tr>
<tr>
<td>0.3</td>
<td>4.63</td>
<td>-0.03</td>
<td>4.67</td>
<td>1.67</td>
<td>14.33</td>
</tr>
</tbody>
</table>
The figure illustrates that second-best solutions are not appropriate if we want to maintain sustainability of firms and society. Trust is important and profitable. Rightfully so, people without trust and fellow-feeling end up in jail, and microeconomists without ethics as well.

**G. Conclusion**

The paper has shown:

1. Trust is an economic variable.
2. There are many mathematical methods that all come to the same conclusion: Trust matters.
3. Mathematical methods prove that trust is essential for the firm’s competitiveness.
4. Mathematical formulations of Aristotle’s ethics and Adam Smith’s “fellow-feeling” prove the importance of Ethics of the Firm as a basis for responsible management.
5. Long-term relationships reduce the moral hazard problem and thereby reduce transaction costs of the firm.
6. Good management means management of trust within the firm and of the long-term relationships with the firm’s shareholders.
7. Management of resilience and sustainability is based on management of trust.
Appendix A: Derivation of Formulae for Aristotle’s Utility Function with Fellow Feeling

Substituting the price (13) and the costs (14) into (11) yields

\[ U(x) = \alpha_1 \left( p(x) x - K(x) \right) + \alpha_2 c x \]
\[ = \alpha_1 \left( A x - B x^2 - C x - D x^2 - FK \right) + \alpha_2 c x. \]  \hfill (17)

Differentiating (17) with respect to \( x \) we find

\[ U'(x) = \alpha_1 \left( A - 2 B x - C - 2 D x \right) + \alpha_2 c \]
\[ = \alpha_1 \left( A - C \right) + \alpha_2 c - 2 \alpha_1 \left( B + D \right) x, \]  \hfill (18)

\[ U''(x) = -2 \alpha_1 \left( B + D \right). \]  \hfill (19)

As our constants \( B \) and \( D \) are assumed to be positive, we see from the sign of the second derivative that any extremum is a maximum. We solve \( U''(x) = 0 \) for \( x \) and find

\[ \alpha_1 \left( A - C \right) + \alpha_2 c = 2 \alpha_1 \left( B + D \right) x \implies x = x^* = \frac{(A - C)}{2(B + D)} + \frac{c}{2 \alpha_1 (B + D)}. \]  \hfill (20)
Hence the price $p^* = p(x^*)$ for $x = x^*$ is given by

$$p^* = p(x^*) = A - B x^* = A - \frac{B (A - C)}{2 (B + D)} - \frac{\alpha_2}{\alpha_1} \frac{B c}{2 (B + D)}. \quad (21)$$

The fellow $F^* = F(x^*)$ feeling for $x = x^*$ is given by

$$F^* = F(x^*) = c x^*, \quad (22)$$

and the profit $P^* = P(x^*)$ for $x = x^*$ is given by

$$P^* = P(p(x^*), x^*) = p(x^*) x^* - K(x^*) = p^* x^* - C x^* - D (x^*)^2 - FK. \quad (23)$$

The utility $U^* = U(x^*)$ for $x = x^*$ is then finally

$$U^* = U(x^*) = \alpha_1 P^* + \alpha_2 F^* \quad (24)$$

subject to $\alpha_1 + \alpha_2 = 1$. We note that, since $\alpha_2 = 1 - \alpha_1$, the factor

$$\frac{\alpha_2}{\alpha_1} = \frac{1 - \alpha_1}{\alpha_1} = \frac{1}{\alpha_1} - 1 \quad (25)$$

is nonlinear in $\alpha_1$. Hence $x^*, p^*, F^*, P^*$ and $U^*$ are also all nonlinear in $\alpha_1$. 
Appendix B: Formulae for Numerical Example

We substitute the numbers for the numerical example

\( C = 2, \ D = 6, \ c = 4, \ B = 1, \ A = 16, \) and \( FK = 4 \)

into the equations (20) to (24) for \( x^*, \ p^*, \ F^*, \ P^* \) and \( U^* \) on the previous page, use

\[
\alpha_2 = 1 - \alpha_1 \quad \text{and} \quad \frac{\alpha_2}{\alpha_1} = \frac{1 - \alpha_1}{\alpha_1} = \frac{1}{\alpha_1} - 1 \quad (26)
\]

and simplify:

\[
x^* = \frac{16 - 2}{2(1 + 6)} + \frac{\alpha_2}{\alpha_1} \frac{4}{2(1 + 6)} = 1 + \frac{\alpha_2}{\alpha_1} \frac{2}{7} = 1 - \frac{2}{7} + \frac{2}{7} \frac{1}{\alpha_1} = \frac{5}{7} + \frac{2}{7} \frac{1}{\alpha_1}, \quad (27)
\]

\[
p^* = 16 - 1 x^* = 16 - \frac{5}{7} - \frac{2}{7} \frac{1}{\alpha_1} = \frac{107}{7} - \frac{2}{7} \frac{1}{\alpha_1}, \quad (28)
\]

\[
F^* = 4 x^* = \frac{20}{7} + \frac{8}{7} \frac{1}{\alpha_1}, \quad (29)
\]

\[
P^* = p^* x^* - 2 x^* - 6 \left( x^* \right)^2 - 4
\]

\[
= \left( \frac{107}{7} - \frac{2}{7} \frac{1}{\alpha_1} \right) \left( \frac{5}{7} + \frac{2}{7} \frac{1}{\alpha_1} \right) - 2 \left( \frac{5}{7} + \frac{2}{7} \frac{1}{\alpha_1} \right) - 6 \left( \frac{5}{7} + \frac{2}{7} \frac{1}{\alpha_1} \right)^2 - 4
\]

\[
= \frac{535}{49} - \frac{10}{49} \frac{1}{\alpha_1} + \frac{214}{49} \frac{1}{\alpha_1} - \frac{28}{49} \frac{1}{\alpha_1} - \frac{28}{49} \frac{1}{\alpha_1} - \frac{150}{49} \frac{1}{\alpha_1} + \frac{120}{49} \frac{1}{\alpha_1} - \frac{24}{49} \frac{1}{\alpha_1} - \frac{196}{49}
\]

\[
= \frac{119}{49} + \frac{56}{49} \frac{1}{\alpha_1} - \frac{28}{49} \frac{1}{\alpha_1} - \frac{17}{7} \frac{1}{\alpha_1} - \frac{8}{7} \frac{1}{\alpha_1} - \frac{4}{7} \frac{1}{\alpha_1^2}, \quad (30)
\]

\[
U^* = \alpha_1 P^* + (1 - \alpha_1) F^* = \frac{17}{7} \alpha_1 + \frac{8}{7} \frac{4}{7} \frac{1}{\alpha_1} + (1 - \alpha_1) \left( \frac{20}{7} + \frac{8}{7} \frac{1}{\alpha_1} \right)
\]

\[
= \frac{17}{7} \alpha_1 + \frac{8}{7} \frac{4}{7} \frac{1}{\alpha_1} + \frac{20}{7} + \frac{8}{7} \frac{1}{\alpha_1} - \frac{20}{7} \frac{1}{\alpha_1} - \frac{8}{7} \frac{4}{7} \frac{1}{\alpha_1} + \frac{8}{7} \frac{1}{\alpha_1} = \frac{20}{7} - \frac{3}{7} \alpha_1 + \frac{4}{7} \frac{1}{\alpha_1}. \quad (32)
\]

So we have found from (27) to (32)

\[
x^* = \frac{5}{7} + \frac{2}{7} \frac{1}{\alpha_1} \approx 0.71 + \frac{0.29}{\alpha_1}, \quad (33)
\]

\[
p^* = \frac{107}{7} - \frac{2}{7} \frac{1}{\alpha_1} \approx 15.29 - \frac{0.29}{\alpha_1}, \quad (34)
\]

\[
F^* = \frac{20}{7} + \frac{8}{7} \frac{1}{\alpha_1} \approx 2.86 + \frac{1.14}{\alpha_1}, \quad (35)
\]

\[
P^* = \frac{17}{7} + \frac{8}{7} \frac{1}{\alpha_1} - \frac{4}{7} \frac{1}{\alpha_1^2} \approx 2.43 + \frac{1.14}{\alpha_1} - \frac{0.57}{\alpha_1^2}, \quad (36)
\]

\[
U^* = \frac{20}{7} - \frac{3}{7} \alpha_1 + \frac{4}{7} \frac{1}{\alpha_1} \approx 2.86 - 0.43 \alpha_1 + \frac{0.57}{\alpha_1}. \quad (37)
\]
Literature


Albach, Horst: Bestimmt die christliche Kinderstube auch heute noch das unternehmerische Handeln? In: Thomas, Hans; Hattler, Johannes (eds.): Ethik im Dienst der Unternehmensführung, Marburg 2008, pp. 73 – 92


Bachmann, Reinhard; Zaheer, Akbar(eds.): Handbook of Trust Research, Cheltenham 2006


Bromiley, Philip; Harris, Jared: Trust, transaction cost economic, and mechanisms, in: Bachmann, R.; Zaheer, A. (eds.): Handbook of Trust Research, 2006


Castaldo, Sandro: Trust in market relationships, Cheltenham 2007

Cook, Karen S.; Hardin, Russell; Levi, Margaret: Cooperation without trust? The Russell Sage Foundation Series on Trust, 2005, number 9


Cremer, David De; Dick, Rolf van; Murnighan, J. Keith: Social Psychology and Organizations, New York – London 2011


Dean, Joel: Capital Budgeting, 3rd edition, New York 1951


Gentz, Manfred; Kirchner, Christian (eds.): Der Preis der Freiheit. Wiedergewinnung von Vertrauen in eine freiheitliche Wirtschafts- und Gesellschaftsordnung, Berlin 28. Oktober 2010 (Economic Society of Berlin Humboldt University)


Gutenberg, Erich: Betriebswirtschaftslehre als Wissenschaft, in: Kölner Universitätsreden Nr. 18, Krefeld 1961

Kayser, Stefan: Die Dynamik der Unternehmenstransformation in Osteuropa, Wiesbaden 1999

Kenning, Peter: Customer Trust Management, ein Beitrag zum Vertrauensmanagement im Lebensmitteleinzelhandel, Wiesbaden 2002


Möllering, Guido: Trust. Reason, Routine, Reflexivity, Bingley 2008


Pawlas, Andreas: Ethik und Marktwirtschaft. Zu den deistischen Grundzügen des marktwirtschaftlichen Konzepts Adam Smiths und ihrer Ambivalenz (Manuskript)


Petersen, Thomas: Optimale Anreizsysteme, in: Beiträge zur betriebswirtschaftlichen Forschung, Band 63, Wiesbaden 1989

Pinkwart, Andreas: Chaos und Unternehmenskrise, in: Beiträge zur betriebswirtschaftlichen Forschung Band 69, Wiesbaden 1992

Schnauss, Martin: Vertrauen und Akzeptanz einer Region als Erfolgsfaktoren für deren Unternehmen, (Manuskript), Zürich 2012


Schoeck, Helmut: Der Neid. Eine Theorie der Gesellschaft, Freiburg/München 1966


Sirdeshmukh, Deepak; Sabol, Barry; Singh, Jagdip: Consumer trust, value and loyalty in relational exchanges, Cambridge, MA, Marketing Science Institute 2001


trust in the Economics topic by Longman Dictionary of Contemporary English | LDOCE | What you need to know about Economics: words, phrases and expressions | Economics. A trust receipt is a legal document that creates a lien on some specific item of inventory. The nurturing and support they received in labor gave them a deep sense of accomplishment and trust in themselves. The Fund has been able to assist with new charitable trusts at Thirlestane and Newliston in Lothian. Wealth Protector, which combines a discretionary trust with a choice of investment plans. This seems to indicate a certain lack of trust in fund managers, rather than a weakness in bibliometric methodology. Don’t preach lack of trust at me. Trust in other people is the glue that holds society together. The erosion of trust coincides with widening disparities in incomes. But does inequality reduce trust? There is evidence that it does, according to research by Eric D. Gould, a professor of economics at Hebrew University, and Alexander Hijzen, a senior economist at the Organisation for Economic Cooperation and Development. They analyzed data from the American National Election Survey from 1980 to 2010. This paper is intended to propose some thoughts about trust and what role it could play in the current European society that has been troubled by economic, political, and social issues. First, I will discuss the present situation in Europe, than I will consider Fukuyama’s studies on trust, and finally I would express my opinion. The 2008 global recession has caused a profound state of tension throughout the world. Talking about Europe, economic crisis, harsh measures, imposed to some countries such as Spain, Italy, Greece, terrorism and mass immigration, have engendered four effects: a) a pove